Quantum and Transport Mobilities of Electrons in GaAs/Ga_{1-x}Al_xAs Multiple Quantum Wells

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The quantum and transport mobilities of electrons in modulation-doped GaAs/Ga_{1-x}Al_xAs multiple quantum wells with well widths in the range between 51 and 145 Å and carrier density of about 1×10^{16} m^{-2} have been investigated by magnetotransport measurements. The magnetic field dependence of the amplitude of the quantum oscillations in both magnetoresistance and Hall resistance have been used to determine the quantum (\tau_q) and transport (\tau_t) lifetimes (and hence the quantum (m_q) and transport (m_t) mobilities) of 2D electrons. The values thus found for \mu_t are substantially smaller than those of the Hall mobility (\mu_H) as obtained in the ohmic regime at low magnetic fields. The discrepancy between \mu_t and \mu_H has been explained in terms of a transport lifetime \tau_t that depends on the electron energy due to the scattering of electrons by interface roughness in the quantum wells.

1. Introduction

In a two-dimensional (2D) electron gas both the magnetoresistance (R_{xx}) and Hall resistance (R_{xy}) oscillate as a function of magnetic field with a periodicity which is determined by the electron density. The theoretical models [1,2] developed for the low-magnetic field limit comprise only one lifetime. In practice, however, it is important to distinguish between the quantum lifetime \tau_q, which is given by the total scattering rate, and the transport lifetime \tau_t, which is weighted by the scattering angle [3 to 7]. Based on the above magnetotransport theories for the 2D electron gas, Coleridge et al. [5] proposed a simple method for the determination of the two lifetimes. The technique involves measurements of the quantum oscillations in both the magnetoresistance and Hall resistance.

At low magnetic fields, the density of states acquires an oscillatory component which can be written as [2,5]

\[
\frac{\Delta g(E)}{g_0} = 2 \sum_{s=1}^{\infty} \exp(-s\pi/\omega_c \tau_q) D(s\chi) \cos\left(\frac{2\pi E}{\hbar \omega_c} - s\chi\right),
\]

where \(g_0\) is the zero-field density of states, \(s\) is the harmonic index, \(\omega_c = eB/m^*\) is the cyclotron frequency, \(m^*\) is the in-plane effective mass of electrons, \(\tau_q\) is the quantum lifetime, and \(E\) is the electron energy. The exponential term, \(\exp(-s\pi/\omega_c \tau_q)\), describes the damping due to the collision broadening of the Landau levels, and

\[
D(s\chi) = \frac{s\chi}{\sinh(s\chi)}
\]
with
\[
\chi = \frac{2\pi^2 k_B T}{\hbar \omega_c}
\]
(3)
describes the temperature damping. In the derivation of Eq. (1) it was assumed that each of the Landau levels can be represented by a Lorentzian with a width \( \Gamma \) independent of energy or magnetic field, such that \( \tau_q = \hbar/2\Gamma \). Assuming that \( E_F \gg \hbar/\tau_q \), \( E_F \gg \hbar \omega_c \) and \( \Delta g(E_F)/g_0 \ll 1 \), and retaining only the fundamental harmonic (with \( s = 1 \)), Coleridge et al. [5] have shown that the oscillatory parts of the diagonal (\( \rho_{xx} \)) and off-diagonal (\( \rho_{xy} \)) components of the resistivity tensor are related by
\[
\frac{1}{2} \frac{\Delta \rho_{xx}}{\rho_0} = -\omega_c \tau_t \frac{\Delta \rho_{xy}}{\rho_0} = \frac{\Delta g(E_F)}{g_0},
\]
(4)
where \( \rho_0 (= \rho_{xx}(B = 0)) \) is the zero-field resistivity, \( E_F \) is the Fermi energy, and \( \tau_t \) is identified [5] as the transport lifetime at zero magnetic field. For a 2D electron gas \( \rho_{xx} = R_{xx} b/L \) and \( \rho_{xy} = R_{xy} \), where \( b \) and \( L \) denote the width and length of the Hall-bar between the voltage contacts, respectively. In Eq. (4) the quantum lifetime appears only through \( \Delta g \), while the transport lifetime \( \tau_t \) appears as a prefactor scaling the oscillations in the Hall resistance. This model has been widely used to determine the quantum lifetime and transport lifetime of 2D electrons in various GaAs/Ga\(_{1-x}\)Al\(_x\)As heterojunctions [5,7 to 13]. Coleridge et al. [5,8] pointed out that the validity of their semiclassical treatment extends into the intermediate magnetic-field regime where \( \omega_c \tau_q < 2 \) and that there is no restriction on the value of \( \omega_c \tau_q \) (see also [9]).

The objective of this work is to determine the quantum and transport mobilities of 2D electrons in modulation-doped GaAs/Ga\(_{1-x}\)Al\(_x\)As multiple quantum wells by measuring the quantum oscillations in both magnetoresistance and Hall resistance. The samples used in the measurements are highly degenerate with approximately equal carrier densities and only the first subband of each sample is populated at liquid-helium temperatures. The data are analysed by using the theoretical model [5] outlined above. The value thus found for the transport mobility (\( \mu_t \)) is compared with that (\( \mu_{1H} \)) obtained from the conventional Hall measurements [14].

### 2. Experimental Procedure

The GaAs/Ga\(_{1-x}\)Al\(_x\)As multiple quantum well (MQW) samples were grown by the MBE technique. The samples were fabricated in Hall-bar geometry (with width \( b = 0.375 \) mm and length \( L = 1.5 \) mm) and ohmic contacts were formed by diffusing Au/Ge/Ni alloy to all the layers. All structures (Fig. 1) were nominally identical except for the quantum well width (\( L_z \)) which was varied from 51 to 145 Å. Table 1 lists the characteristics of the samples.

The magnetoresistance and Hall resistance were measured simultaneously as a function of magnetic field \( B \) (0 to 2.3 T) at liquid-helium temperatures (1.5 to 4.2 K). In all cases the dc current (\( I \)) through the sample was low enough to avoid carrier heating [15]. The current flow was in the plane of the MQW and the magnetic field was perpendicular to the plane of the sample and hence to the 2D electron gas. The current source
and the voltmeter (which was used to measure the voltages \( V_{xx} \) and \( V_{xy} \) corresponding to the magnetoresistance and the Hall resistance, respectively) were controlled by a personal computer and the data were collected automatically. The data were taken at equal intervals of \( 1/B \).

### 3. Results and Discussion

Both, the magnetoresistance \( R_{xx} (= V_{xx}/I) \) and Hall resistance \( R_{xy} (= V_{xy}/I) \) for all the samples exhibited quantum oscillations. As an example, Fig. 2 shows \( R_{xx} \) and \( R_{xy} \) versus magnetic field for a GaAs/Ga_{1-x}Al_{x}As MQW sample with well width of 145 Å. In the range of magnetic fields used in the measurements, well-resolved steps were observed in the Hall resistivity varying by nearly \((h/e^2)/2iN_w\), where \( N_w = 10 \) is the number of quantum wells and \( i \) is the Landau index [16]. This demonstrates that all of the wells in the MQW are connected in parallel and all of them contributed to the quantized conditions. The oscillations in \( R_{xx}(B) \) are superimposed on a monotonic background which differs from one sample to another [14,15]. The oscillating components \( \Delta R_{xx} \) and \( \Delta R_{xy} \) in the magnetoresistance and Hall resistance, respectively, are obtained by fitting the

<table>
<thead>
<tr>
<th>Sample</th>
<th>( x ) (%)</th>
<th>Well width ( L_z ) (Å)</th>
<th>First subband energy ( E_1 ) (meV)</th>
<th>2D electron density ( n_{2D} ) ( \times 10^{16} ) m(^{-2})</th>
<th>Hall density ( n_{H} ) ( \times 10^{17} ) m(^{-2})</th>
<th>Electron effective mass ( m^* ) ( (m_0) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>C579</td>
<td>32</td>
<td>51</td>
<td>73.7</td>
<td>0.99</td>
<td>0.94</td>
<td>0.0746</td>
</tr>
<tr>
<td>ES2</td>
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<td>45.5</td>
<td>1.08</td>
<td>1.17</td>
<td>0.0725</td>
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<tr>
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<td>32</td>
<td>78</td>
<td>43.1</td>
<td>1.11</td>
<td>1.07</td>
<td>0.0717</td>
</tr>
<tr>
<td>C568</td>
<td>32</td>
<td>106</td>
<td>27.7</td>
<td>1.12</td>
<td>1.09</td>
<td>0.0655</td>
</tr>
<tr>
<td>C580</td>
<td>32</td>
<td>145</td>
<td>16.9</td>
<td>1.05</td>
<td>1.18</td>
<td>0.0657</td>
</tr>
</tbody>
</table>

Table 1: Sample parameters of the modulation-doped GaAs/Ga_{1-x}Al_{x}As multiple quantum wells. The data for the effective mass are taken from [15].
Fig. 2. Magnetoresistance ($R_{xx}$) and Hall resistance ($R_{xy}$) as a function of magnetic field for sample C580 measured at 4.2 K.

Fig. 3. Oscillating components of the a) magnetoresistance ($\Delta R_{xx}$) and b) Hall resistance ($\Delta R_{xy}$) obtained by subtracting the nonoscillating components from the raw experimental data given in Fig. 2.
nonoscillatory component to a polynomial of second degree, and then subtracting it from the raw experimental data. The oscillations in both $\Delta R_{xx}(B)$ and $\Delta R_{xy}(B)$ are sinusoidal with well-defined envelopes and are almost symmetrical about a horizontal line (Fig. 3). For each sample the periods of the oscillations in $\Delta R_{xx}(B)$ and $\Delta R_{xy}(B)$ are equal within the experimental error. However, the amplitudes of the Shubnikov-de Haas (SdH) oscillations in the magnetoresistance (Fig. 3a) are about an order of magnitude larger than those in the Hall resistance (Fig. 3b). The quantum oscillations observed in $\Delta R_{xx}$ and $\Delta R_{xy}$ are in antiphase as predicted theoretically [5]. The Fourier analysis of the SdH oscillations confirms that, for each sample studied, only the first subband is populated and that the contribution of higher harmonics (with $s \geq 2$) is negligible. The fact that the SdH oscillations contains only one period also implies that all the parallel connected quantum wells exhibit almost the same electron density in the populated ground state. The 2D electron density (or the sheet density per quantum well), $n_{2D}$, as determined from the period of the SdH oscillations is found to have the same value within 10% for all the samples as shown in Table 1. The sheet electron density, $n_H$, obtained from Hall measurements is also given in Table 1. In a MQW sample with ten identically doped quantum wells, the sheet electron density is expected to be exactly a factor of ten higher than the 2D density. In all our samples the ratio $n_H/n_{2D}$ is about ten within the experimental accuracy indicating that the 2D electron densities are identical in all the quantum wells in each sample and only these electrons contribute to transport, thus, parallel conduction [17] is negligible.

The $\Delta R_{xx}$ and $\Delta R_{xy}$ data at each oscillation extremum and the values [15] for the in-plane effective mass ($m^*$) of 2D electrons have been used to calculate the magnitude of the reduced resistivities $\frac{1}{2D(\chi)}(\Delta \rho_{xx}/\rho_0)$ and $\frac{\omega_e \tau_1}{D(\chi)}(\Delta \rho_{xy}/\rho_0)$. Inspection of Eqs. (1) and (4) reveals that the logarithm of these reduced resistivities should be linear in reciprocal magnetic field with a slope given by $-\pi m^*/e\tau_0$ and an intercept of $\ln 2$ as

\[ \ln \left[ \frac{\rho_{xx}(2D(\chi))}{\rho_0} \right], \ln \left[ \frac{\rho_{xy}(2D(\chi))}{\rho_0} \right] \]

\[ -0.2 \]

\[ -0.4 \]

\[ 0 \]

\[ 0.2 \]

\[ 0.4 \]

\[ 0.6 \]

\[ 0.8 \]

\[ 1/ B ( T^{-1}) \]

**Fig. 4.** Determination of the quantum and transport lifetimes. The data points represented by the solid squares and open circles correspond to reduced resistivities $\frac{1}{2D(\chi)}(\Delta \rho_{xx}/\rho_0)$ and $\frac{\omega_e \tau_1}{D(\chi)}(\Delta \rho_{xy}/\rho_0)$, respectively. The straight line is the least-squares fit to the magnetoresistance data only.
Fig. 4 shows the natural logarithm of both reduced resistivities as a function of $1/B$ (i.e., the Dingle plot) for the MQW sample with $L_z = 145 \text{ Å}$. The straight line in the figure represents the least-squares fit to the magnetoresistance data only, the slope of which is used to determine the quantum lifetime $t_q$ of 2D electrons (Table 2). We note that, for each MQW sample studied, the intercept at $1/B = 0$ of the Dingle plot is found to be much smaller (Table 2) than predicted theoretically, and that the values obtained for $t_q$ are somewhat different from those reported by Çelik et al. [15] where slightly different Dingle plots were used. Equation (4) indicates that the magnitudes of the two reduced resistivities at an oscillation extremum must be equal if an appropriate value is assigned to the transport lifetime $t_t$. Accordingly, the value of $t_t$ is chosen such that the magnetic field dependence of $\ln \Omega_{cy}/\Omega_0$ fits that of $\ln [(1/2D(\chi)) (\Delta Q_{xy}/Q_0)]$ (see Fig. 4).

The values obtained for $t_q$ and $t_t$ of 2D electrons in the GaAs/Ga$_{1-x}$Al$_x$As MQWs are compared in Table 2. The transport lifetime is larger than the quantum lifetime by a factor of about 2 to 3 for all the samples except the one with $L_z = 145 \text{ Å}$ for which the two lifetimes are found to be almost equal. Theoretical calculations relating the quantum lifetime to the transport lifetime predict a $t_q/t_t$ ratio of less than unity for small-angle scattering and equal to or greater than unity for wide-angle scattering in the extreme quantum limit for single subband occupancy (see for instance [3,4,7]).

The values found for the transport mobility $\mu_t$ are by a factor of 1.5 and 3 lower than the Hall mobility $\mu_H$ (Fig. 5). One reason for the discrepancy might be associated with parallel conduction that occurs in layered structures, like the multiple quantum wells investigated in this work, where undepleted electrons in the barrier regions (low mobility channel), separating the quantum wells (high mobility channel), can contribute to the conductivity [17]. In crossed electric and magnetic fields (Hall effect) the net effect of parallel conduction is a magnetic field dependent Hall mobility that is

<table>
<thead>
<tr>
<th>Sample</th>
<th>$L_z$ (Å)</th>
<th>$t_q$ (10^{-12}s)</th>
<th>$t_t$ (10^{-12}s)</th>
<th>$\mu_q$ (m² V⁻¹ s⁻¹)</th>
<th>$\mu_t$ (m² V⁻¹ s⁻¹)</th>
<th>$\mu_H$ (m² V⁻¹ s⁻¹)</th>
<th>Intercept</th>
</tr>
</thead>
<tbody>
<tr>
<td>C579</td>
<td>51</td>
<td>0.17</td>
<td>0.53</td>
<td>0.40</td>
<td>1.25</td>
<td>2.43</td>
<td>−0.27</td>
</tr>
<tr>
<td>ES2</td>
<td>75</td>
<td>0.26</td>
<td>0.52</td>
<td>0.63</td>
<td>1.26</td>
<td>2.98</td>
<td>−0.52</td>
</tr>
<tr>
<td>C581</td>
<td>78</td>
<td>0.21</td>
<td>0.44</td>
<td>0.51</td>
<td>1.08</td>
<td>3.75</td>
<td>−0.35</td>
</tr>
<tr>
<td>C568</td>
<td>106</td>
<td>0.15</td>
<td>0.30</td>
<td>0.40</td>
<td>0.80</td>
<td>2.42</td>
<td>−0.03</td>
</tr>
<tr>
<td>C580</td>
<td>145</td>
<td>0.39</td>
<td>0.37</td>
<td>1.04</td>
<td>0.99</td>
<td>2.58</td>
<td>−0.78</td>
</tr>
</tbody>
</table>

$1/B \to 0$ (see [5,7,8]).
different from transport mobility. However, at non-zero magnetic fields, the transport properties of the layered structure are expected to be dominated by the lower mobility channel as given by Eqs. (9) to (14) in [17]. Therefore, one would expect a reduction in the Hall mobility not the observed increase (Fig. 5). Besides the ratio of about ten between the measured values of Hall and 2D electron densities (Table 1) indicates strongly that parallel conduction can be ruled out.

We believe that the reason for the observed discrepancy between the transport and Hall mobilities is simply the Hall scattering factor $r$ [19]. It is well known that, in a highly-degenerate bulk material where lattice scattering is dominant $r$ is almost unity. It can, however, be as high as three in materials where ionized impurity scattering is dominant and the scattering time ($\tau_i$) depends on the kinetic energy of the electron. The Hall scattering time ($\tau_{ii}$), which is the energy averaged value of the scattering time $\tau_i$, is therefore $\tau_{ii} = r \tau_i$ and hence $\mu_{ii} = r \mu_i$. If the scattering time has an energy dependence of the form $(\tau_i)^{-1} \propto E^l$ where $l > 1$, then $r > 1$. This is the case for impurity scattering where $l > 3/2$ [19].

In heavily modulation-doped quantum wells with undoped spacer layers, ionized impurity scattering of electrons in the wells is negligible [18]. Therefore, the observation that $r > 1$ cannot be attributed to ionized impurity scattering in our MQW samples. Our previous studies on scattering mechanisms in MQWs indicated strongly that the momentum relaxation of 2D electrons in these structures is determined, predominantly, by interface roughness scattering [15,18]. In this process, the scattering potential $W_M$ is assumed to be due to well width fluctuations in the form of multiples of monolayers ($\Delta$) with a lateral length having a Poisson distribution around a mean value $\lambda$. At low enough temperatures ($kT \ll W_M$) and for electron energies ($E$) much smaller than $W_M$, ($E \leq 0.1 W_M$), the scattering rate is expected to depend on energy and increases with increasing energy similar to the case in neutral impurity scattering [20]. Therefore, in

![Fig. 5. Low-temperature electron mobilities vs. quantum well width of GaAs/Ga$_{1-x}$Al$_x$As multiple quantum wells. The solid squares, open triangles, and solid circles correspond to the quantum mobility ($\mu_q$), the transport mobility ($\mu_t$), and the Hall mobility ($\mu_{ii}$), respectively](image-url)
the relationships $\tau_H = rt$ and $\mu_H = rm$, $r$ is expected to be greater than unity as observed in our experiments (Fig. 5). The exact value of $r$ obviously depends on the particulars of the interface roughness parameters in the individual MQW samples.

4. Conclusions

The low-temperature quantum ($\mu_q$) and transport ($\mu_t$) mobilities of electrons in GaAs/Ga$_{1-x}$Al$_x$As MQWs have been determined by measuring the quantum oscillations in magnetoresistance and Hall resistance. The data have been analyzed using the theory by Coleridge et al. [5]. The values found for $\mu_t$ are significantly smaller than those of the Hall mobility ($\mu_H$) obtained in the ohmic regime at low magnetic fields. The intercept at $1/B = 0$ of the Dingle plot for each MQW sample is found to be much smaller than that predicted theoretically. It is shown that the experimental technique employed is sensitive enough to give information about interface roughness scattering even in highly degenerate 2D structures. Our current results confirm earlier independent conclusions that the momentum relaxation in GaAs/Ga$_{1-x}$Al$_x$As multiple quantum wells is limited mainly by interface roughness scattering.

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References